Quantum Networking

Abstract

We aim to optimize the routing strategy and costs via cost vector analysis in quantum networks using the QuNet package. In Quantum networks the act of purifying multiple entanglement states into one single state with higher fidelity has no counter part in classical networking, therefore we employ "multi-path entanglement routing" and analyze the trade of between different Quantum process.

Mathematical Background

Any network can be thought as graph

$$G = (V, E)$$

where V is the number of nodes/end-users which can perform any quantum processes like swapper or router etc. Edges E indicates the quantum communication channel.

Entanglement has proven to be great resource for many fundamental processes. For qubit based-networks, it is advantageous to use maximally entangled bell pairs. These bell pairs bypass the no-cloning limitaions and easier to purify due to symmetric nature.

$$|\phi^{\pm}\rangle_{A,B} = \frac{1}{\sqrt{2}}(|0\rangle_A\,|1\rangle_B \pm |1\rangle_A\,|0\rangle_B\,)$$

$$|\psi^{\pm}\rangle_{A,B} = \frac{1}{\sqrt{2}}(|0\rangle_A\,|0\rangle_B \pm |1\rangle_A\,|1\rangle_B\,)$$

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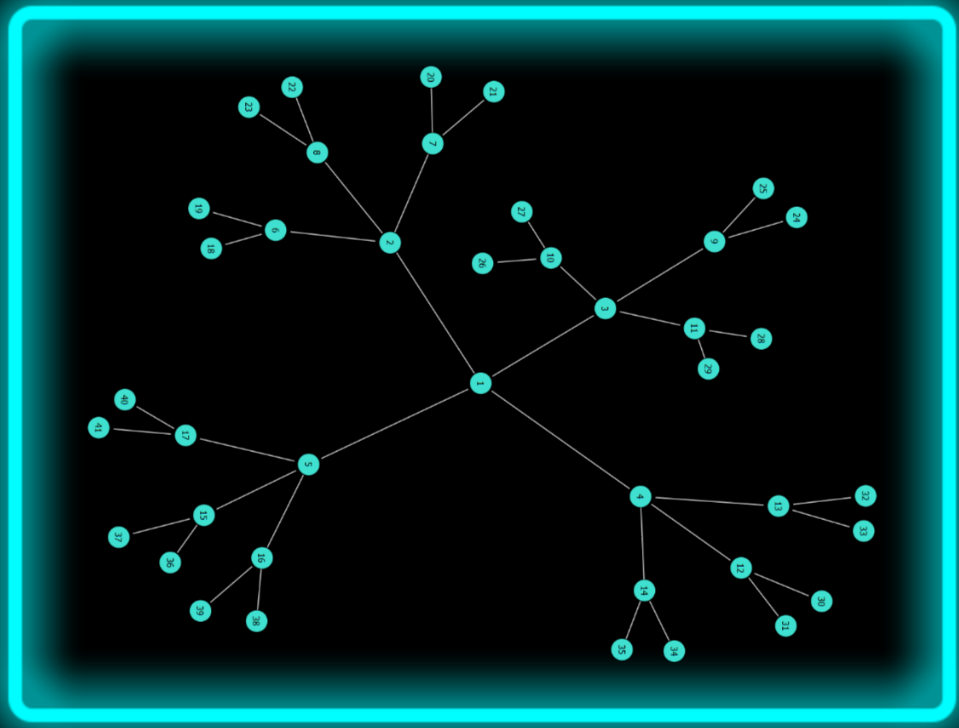

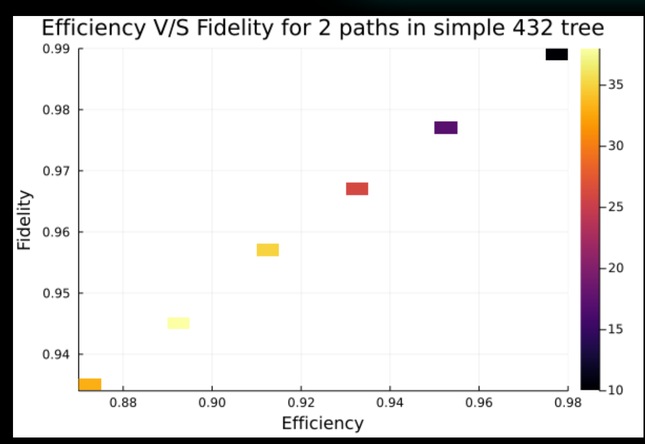

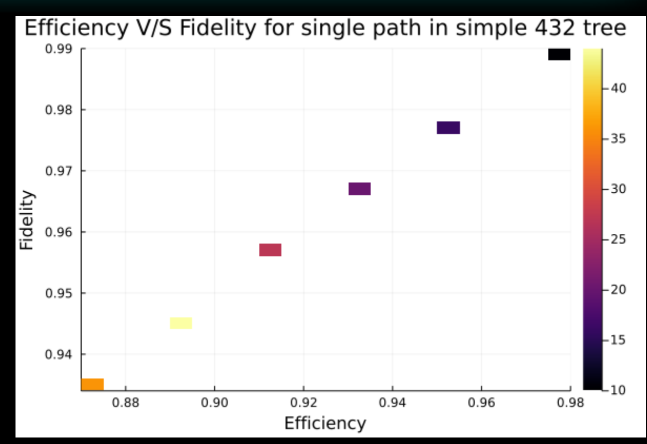

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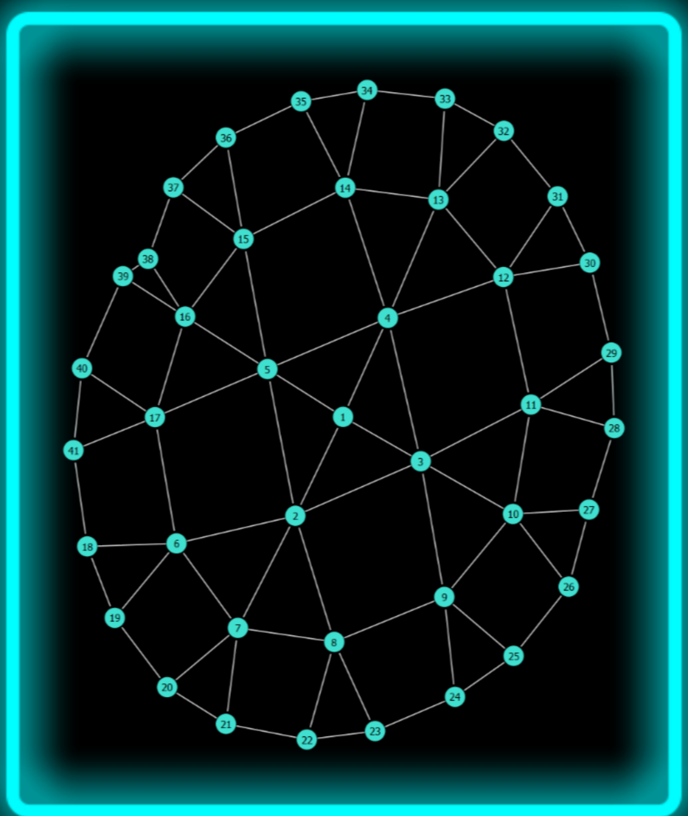

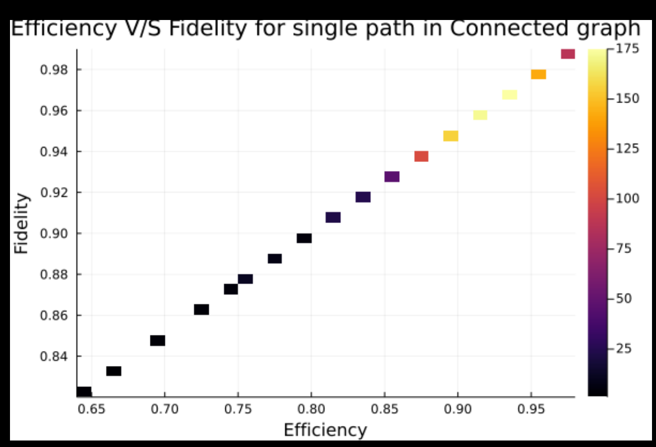

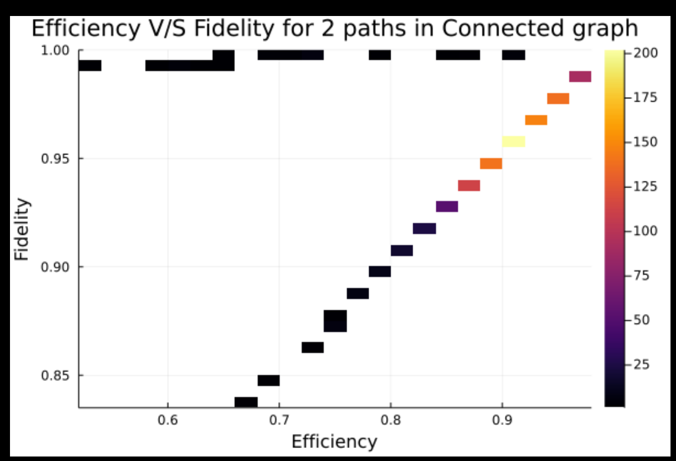

Quantum Channels

A quantum channel provides link between two nodes(user pair) on graph and is subject to quantum processes like decoherence and loss.

$$\varepsilon_{loss}(\hat{\rho}) = p\hat{\rho} + (1-p)|vac\rangle\langle vac|$$

$$\varepsilon_{depolarization}(\hat{\rho}) = p\hat{\rho} + (1-p)\frac{\hat{\mathbb{I}}}{2}$$

$$\varepsilon_{dephasing}(\hat{\rho}) = p\hat{\rho} + (1-p)\hat{Z}\hat{\rho}\hat{Z}$$

$$= (2p-1)\hat{\rho} + (1-p)(\hat{Z}\hat{\rho}\hat{Z} + \hat{\rho})$$

Cost Vector Analysis and Routing Strategies

In Graphs, edges weighted by their respective costs that obey additivity. Cost of each edge in the route is added to yield the net cost of the path. In Quantum process things like probability are multiplicative rather than additive. therefore

$$p_{net} = \prod_i (2p_i - 1) \quad \Rightarrow \quad \log P_{net} = \sum_i \log(2p_i - 1)$$

For single path routing QuNet package make use of the Dijkstra's shortest path algorithm, which takes edge costs into account and returns the path with least cost. In multi-path routing it makes use of the entire network on the premise that qubits from different routes can be purified to get a single a qubit to end user with higher quality.

Till Date and To Do's

All the graphs and heatmaps are generated using QuNet package in Julia language. we put multi_path routing and purifying algorithms used in QuNet to test on simple tree graphs. we analyzed the tradeoff between efficiency and fidelity of bell sates in different types of graphs with different number of allowed paths per user.

1. The graph shown is simple tree graph with branches of 4,3 and 2 at depths 1,2 and 3 respectively. The heatmaps below the graphs shows a linear relation between efficiency and fidelity for both single path and 2 paths, that is because in tree graphs there is only one path for each user to communicate with other user. The results are as expected
2. Graph here is different from previous one because this has many redundant edges between any two nodes that allows the multi-path routing. The heatmaps shows that straight line for single path but for 2 paths we see extra point with high values of fidelity along with the linear line. The results are expected.
3. Third graph has lower number of redundant paths yet enough to allow multi-path routing. We expected the efficiency to decrease when the redundant edges decrease, but a generalization of multi-path routing is multi-user routing. In this scheme, graph is partitioned such that union of all sub graphs make the network. Thus an increase in efficiency is observed.

Research is under way to check the effects of edge redundancy on a given graph to optimize the QNetworks.

References

1. QuNet: Cost vector analysis & multi-path entanglement routing in quantum networks, H. Leone, N. R. Miller, D. Singh, N. K. Langford, P.P. Rohde.(May,2021)
2. Rohde, P. (2021). *The Quantum Internet: The Second Quantum Revolution*. Cambridge: Cambridge University Press. doi:10.1017/9781108868815

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