Thermal Entanglement in two superconducting qubits

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Abstract

We investigate the thermal entanglement in two superconducting qubits. We calculate the concurrence of the system to quantify the thermal entanglement. We suggest a scheme, where an external tunable coupler qubit sandwich between two superconducting qubits generates entanglement. The behavior of concurrence is analyzed for three different cases, in which we consider the effects of the temperature, the qubit-qubit effective coupling strength, and the qubit frequencies on the entanglement. We find that to achieve maximally entangled states, two qubits must have same frequencies.

Model

- The energy of the circuit elements is defined as

$$E(t) = \int_{-\infty}^{t} V(t')I(t')dt'$$

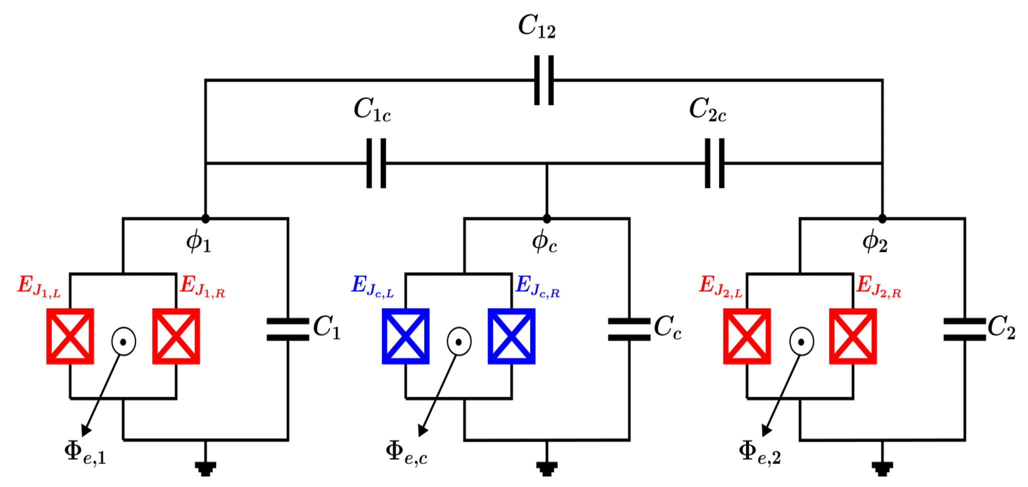
- $I = C\frac{dV}{dt}$
- $V = \frac{d\phi}{dt}$
- $I = I_c sin\phi$

$$\hat{H} = 4E_{C_1}\hat{n}_1^2 + 4E_{C_2}\hat{n}_2^2 + 4E_{C_c}\hat{n}_c^2 - E_{J_1}\cos\hat{\phi}_1 - E_{J_c}\cos\hat{\phi}_c - E_{J_2}\cos\hat{\phi}_2 + 8\frac{C_{1c}}{\sqrt{C_1C_c}}\sqrt{E_{C_1}E_{C_c}}(\hat{n}_1\hat{n}_c) + 8\frac{C_{2c}}{\sqrt{C_2C_c}}\sqrt{E_{C_2}E_{C_c}}(\hat{n}_2\hat{n}_c) + 8(1+\eta)\frac{C_{12}}{\sqrt{C_1C_2}}\sqrt{E_{C_1}E_{C_2}}(\hat{n}_1\hat{n}_2)$$

- $\hat{n} = \frac{i}{2}\sqrt[4]{\frac{E_J}{2E_C}}(\hat{b}^\dagger - \hat{b})$
- $\hat{\phi} = \sqrt[4]{\frac{2E_C}{E_J}}(\hat{b}^\dagger + \hat{b})$

$$\hat{H} = \hat{H}_1 + \hat{H}_c + \hat{H}_2 + \hat{H}_{1c} + \hat{H}_{2c} + \hat{H}_{12}$$
$$\hat{H}_\lambda = \omega_\lambda \hat{b}_\lambda^\dagger \hat{b}_\lambda + \frac{\alpha_\lambda}{2}\hat{b}_\lambda^\dagger \hat{b}_\lambda^\dagger \hat{b}_\lambda \hat{b}_\lambda, \quad \lambda \in \{1, c, 2\}$$
$$\hat{H}_{jc} = g_j(\hat{b}_j^\dagger \hat{b}_c + \hat{b}_j \hat{b}_c^\dagger - \hat{b}_j^\dagger \hat{b}_c^\dagger - \hat{b}_j \hat{b}_c), \quad j = 1, 2$$
$$\hat{H}_{12} = g_{12}(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_1 \hat{b}_2^\dagger - \hat{b}_1^\dagger \hat{b}_2^\dagger - \hat{b}_1 \hat{b}_2)$$

- $\omega_\lambda = \sqrt{8E_{J_\lambda}E_{C_\lambda}} - E_{J_\lambda}$ defines the oscillator frequency.
- $g_j = \frac{1}{2}\frac{C_{jc}}{\sqrt{C_jC_c}}\sqrt{\omega_j\omega_c}$ and $g_{12} = \frac{1}{2}(1+\eta)\frac{C_{12}}{\sqrt{C_1C_2}}\sqrt{\omega_1\omega_2}$ expresses qubit-coupler and qubit-qubit coupling strength, respectively.



Schematic diagram of coupled qubits. Here, C_λ and C_{jc} describe the dominant mode capacitance and the coupling capacitance between qubit j and coupler, respectively. C_{12} is the direct coupling capacitance between the qubits.

SCHRIEFFER-WOLFF TRANSFORMATION

- To decouple the coupler from the system [1]

$$\tilde{H} = e^{-S}He^{S} = H + [H, S] + [[H, S], S]/2 + \ldots$$

with

$$\hat{S} = exp\left\{\sum_{j=1,2}\left[\frac{g_j}{\Delta_j}(\hat{b}_j^\dagger \hat{b}_c - \hat{b}_j \hat{b}_c^\dagger) - \frac{g_j}{\sum_j}(\hat{b}_j^\dagger \hat{b}_c^\dagger - \hat{b}_j \hat{b}_c)\right]\right\}$$

- $\Delta_j = \omega_j - \omega_c$
- $\sum_j = \omega_j + \omega_c$

$$\tilde{H} = \tilde{\omega}_j \hat{b}_j^\dagger \hat{b}_j + \frac{\tilde{\alpha}_j}{2}\hat{b}_j^\dagger \hat{b}_j^\dagger \hat{b}_j \hat{b}_j + \tilde{g}(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_1 \hat{b}_2^\dagger)$$

$$\tilde{\omega}_j \approx \omega_j + g_j^2\left(\frac{1}{\Delta_j} - \frac{1}{\sum_j}\right); \quad \tilde{\alpha}_j \approx \alpha_j$$

$$\tilde{g} \approx \frac{g_1g_2}{2}\left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2} - \frac{1}{\sum_1} - \frac{1}{\sum_2}\right) + g_{12}$$

- The Hamiltonian of coupled superconducting qubits can be written in terms of Pauli operators as [2]

$$\tilde{H} = \sum_{j=1,2}\frac{1}{2}\tilde{\omega}_j\sigma_j^z + \tilde{g}(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+)$$

- In the standard two-qubit basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$, the Hamiltonian of the system can be expressed as

$$\tilde{H} = \begin{pmatrix} \frac{\alpha}{2} & 0 & 0 & 0 \\ 0 & \frac{\tilde{\omega}}{2} & \tilde{g} & 0 \\ 0 & \tilde{g} & -\frac{\tilde{\omega}}{2} & 0 \\ 0 & 0 & 0 & -\frac{\alpha}{2} \end{pmatrix}$$

Thermal Entanglement

- The eigenvectors of the Hamiltonian are given as:

$$|\phi_1\rangle = |0,0\rangle, \quad |\phi_2\rangle = |1,1\rangle$$
$$|\phi_3\rangle = \frac{1}{\sqrt{1+\frac{\xi^2}{4\tilde{g}^2}}}\left(\frac{\xi}{2\tilde{g}}|1,0\rangle + |0,1\rangle\right)$$
$$|\phi_4\rangle = \frac{1}{\sqrt{1+\frac{\zeta^2}{4\tilde{g}^2}}}\left(\frac{\zeta}{2\tilde{g}}|1,0\rangle + |0,1\rangle\right)$$

- The thermal density matrix $\rho(T)$ for this system in terms of standard basis can be written as [3]

$$\rho(T) = \frac{e^{-\beta H}}{Z} = \frac{1}{Z}\sum_{i=1}^{4} e^{-\beta E_m}|\psi_i\rangle\langle\psi_i|$$

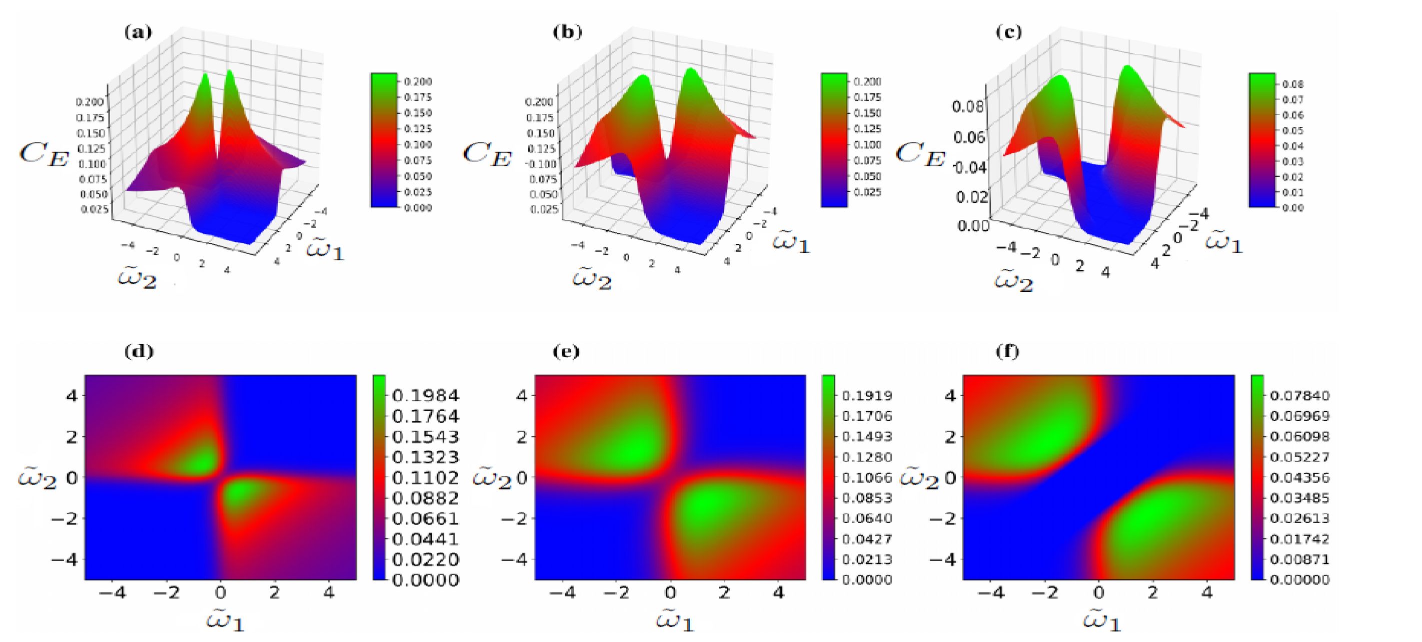
$$\rho(T) = \frac{1}{Z}\begin{pmatrix} \rho_{11} & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & \rho_{44} \end{pmatrix}$$

- The partition function Z is calculated as

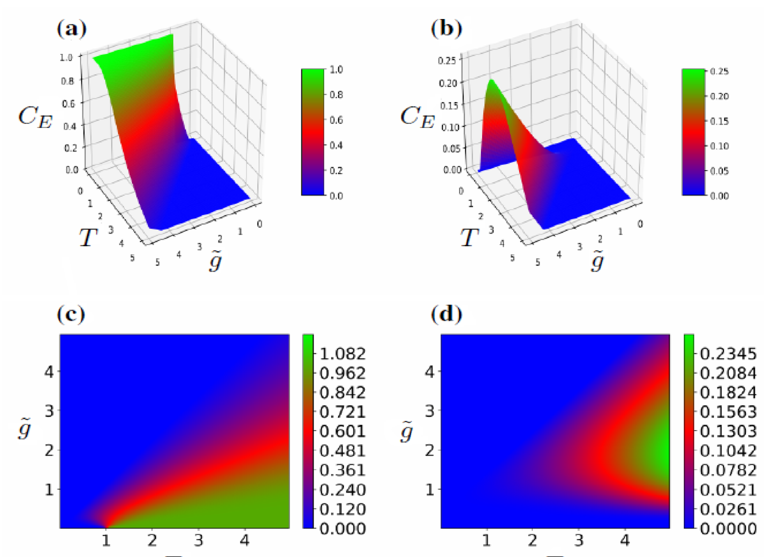
$$Z = tre^{-\beta H} = 2\left[\cosh\left(\frac{\alpha}{2T}\right) + \cosh\left(\frac{\gamma}{2T}\right)\right]$$

- The concurrence 0 corresponds to separable or unentangled states and 1 defines maximally entangled states and is expressed as [4]

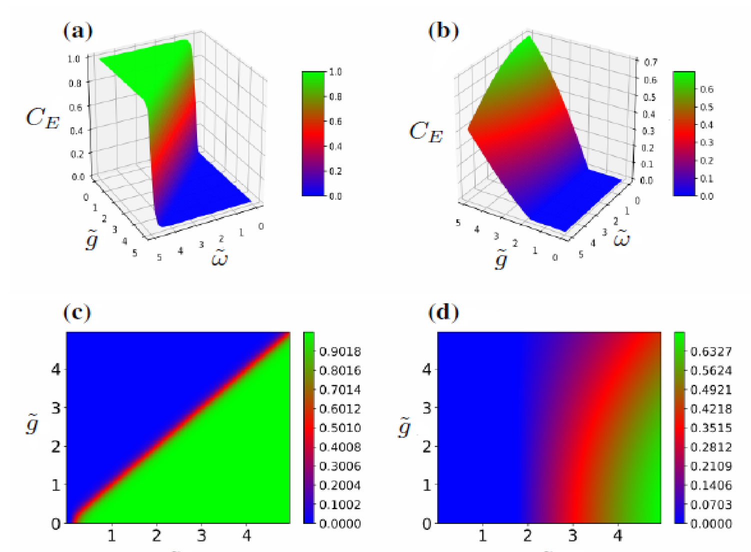
$$C_E = max[0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4]$$



The two-dimensional concurrence in two coupled qubit system is plotted versus dimensionless frequencies $\tilde{\omega}_1$ and $\tilde{\omega}_2$ for (a,d) $\tilde{g} = T = 0.4$, (b,e) $\tilde{g} = T = 0.2$ and (c,f) $\tilde{g} = 0.2$ & $T = 0.4$. T is plotted in units of Boltzmann's constant where $k_B = 1$.



The two-dimensional concurrence is plotted versus T and $\tilde{g}$ for two coupled qubits. The left panel corresponds to $\tilde{\omega}_1 = \tilde{\omega}_2 = 1$ case and right panel corresponds to $\tilde{\omega}_1 = 5$ and $\tilde{\omega}_2 = 1$ case.



The two-dimensional concurrence in two coupled qubit system is plotted versus $\tilde{g}$ and $\tilde{\omega}$ for different values of dimensionless temperature (a,c) $T = 0.1$ and for (b,d) $T = 2$.

Conclusion

- We have found that the thermal entanglement can be efficiently controlled through the effective qubit-qubit coupling strength, qubit frequencies, and temperature.
- The effects of effective qubit-qubit coupling strength and temperature on entanglement are studied for different values of qubit frequencies. We observed that entanglement exists and it can be enhanced by using the coupled qubits with the same frequencies.
- Our results imply that the two coupled qubits can go to maximally entangled states at low temperature and for equal and high values of qubit frequencies.
- This model can also be beneficial for study thermal entanglement for many ($n > 2$) qubit states. Our findings identify the real potential to study thermal entanglement, especially for superconducting qubits.

References

[1] F. Yan et al., Physical Review Applied **10**, 054062 (2018).
[2] J. Kelly et al., Nature **519**, 66 (2015).
[3] H. Bernien et al., Nature **551**, 579 (2017).
[4] C. Song et al., Phys. Rev. Lett. **119**, 180511(2017).

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