Information transfer with Continuous Time Quantum Walks

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Abstract

The ways in which continuous time quantum walks (CTQW) can be directed are explored in this project. It is demonstrated for certain graphs that one can direct, suppress, and enhance CTQW towards a specific site via introducing Gaussian wave packets, linear ramp with complex edge weights, and changing real valued edge weights. CTQW in open quantum systems is also explored using Lindblad operators.

Keywords: Continuous time quantum walks, directionality, enhancement, suppression

Methods

Eq(3) is generalization of eq(1) also known as the Lindblad-Kossakowski equation[1]

$$\dot{\rho}(t) = -i\,[H,\rho] + \sum_k L_k L_k^\dagger - \frac{1}{2}\left(L_k^\dagger L_k \rho + \rho L_k^\dagger L_k\right) \tag{3}$$

Where ρ is the density matrix and L_k are the Lindbald operators representing interaction with the environment. For example:

- Dephasing Operator: $L_{\phi,n} = |n\rangle\langle n|$
- Recombination Operator: $L_{\gamma,n} = |d\rangle\langle n|$
- Trap Operator: $L_{\tau,n} = |\tau\rangle\langle n|$

Where n = vertex, d = recombination site, and τ = trap site. The probability of the walk being at vertex n after time t is:

$$P_n = \langle n|\rho(t)|n\rangle \tag{4}$$

Eq(3) is numerically solved using Euler method and probability for each vertex over time is plotted for two main graphs under different conditions. One is a branched graph[3] and one is a ring[4].

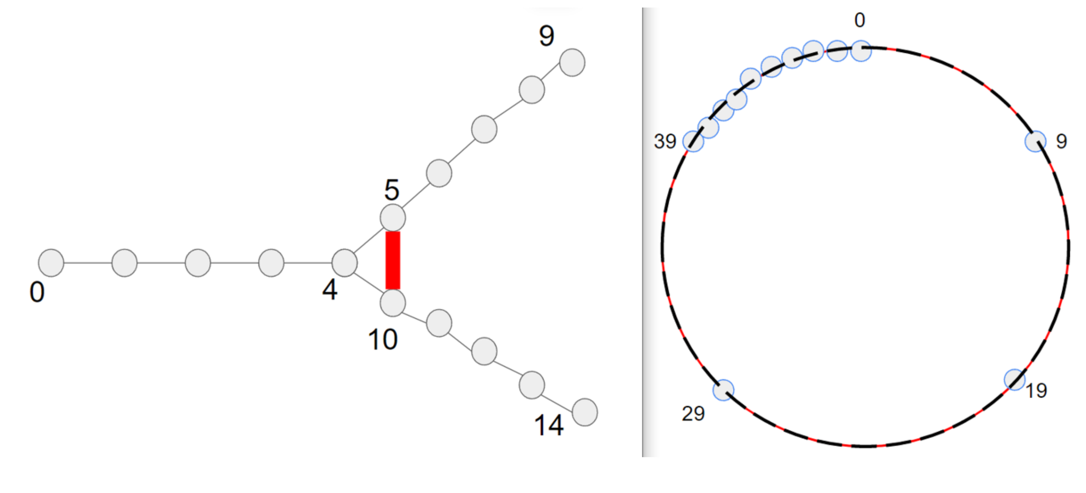

Figure 1: Graphs

Introduction

CTQW[2] takes place on a graph represented as an N-dimensional Hilbert space (N = no. of vertices) and is governed by the following equation:

$$i\frac{d\langle a \mid \psi(t)\rangle}{dt} = \sum_b \langle a|H|b\rangle\langle b \mid \psi(t)\rangle \tag{1}$$

Where $|a\rangle$ denotes the graph's vertex (or node) a, $|\psi(t)\rangle$ is the state vector, and H is the Hamiltonian (formed from Adjacency matrix of the graph). The walk evolves as $|\psi(t)\rangle = \exp^{-i\hat{H}t}|\psi(0)\rangle$, where $|\psi(0)\rangle$ is the initial state of the walker. The probability of being on vertex n after time t is:

$$P_n = |\langle n|e^{-iHt}|\psi(0)\rangle|^2 \tag{2}$$

Results 1

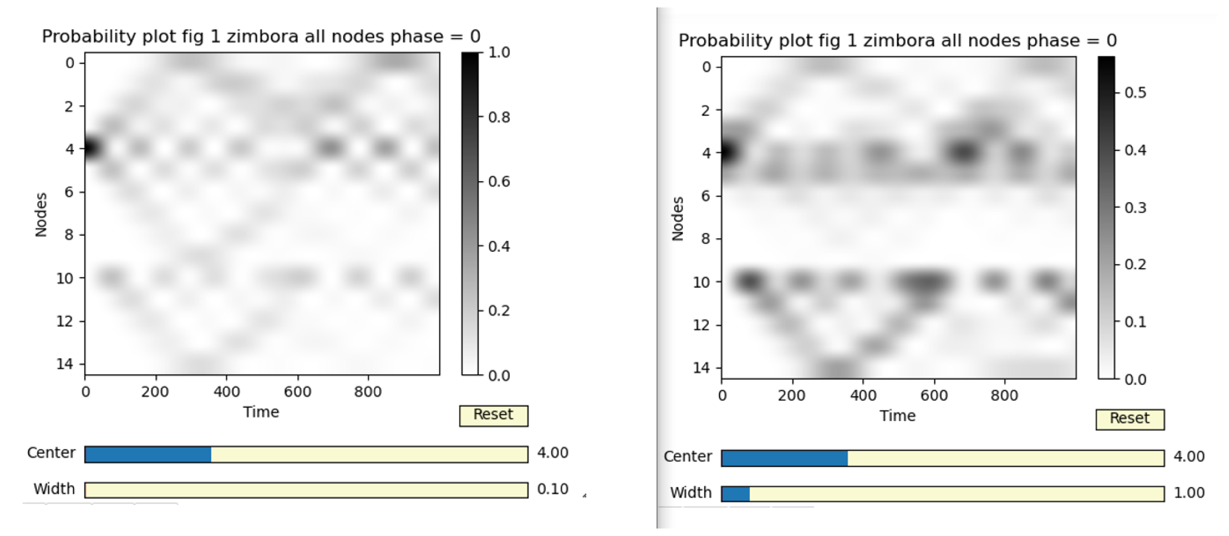

Figure 2: Walk on branched graph with initial Gaussian width=0.1 (left) & 1.0 (right), center=4, red edge phase=0

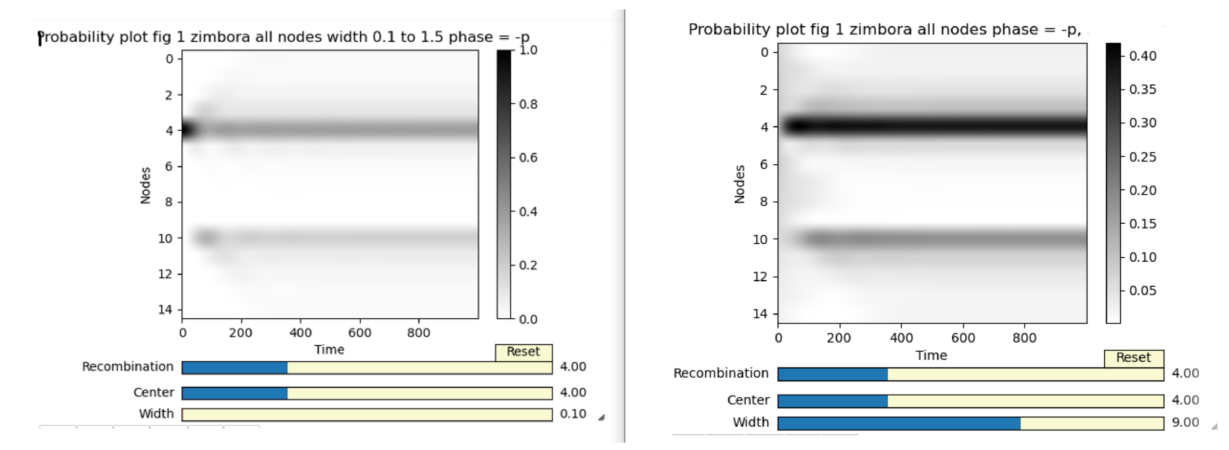

Figure 3: Walk on branched graph with initial Gaussian width=0.1(left) & 9.0 (right), center=4, recombination site=4, red edge phase = -$\pi/2$

Results 2

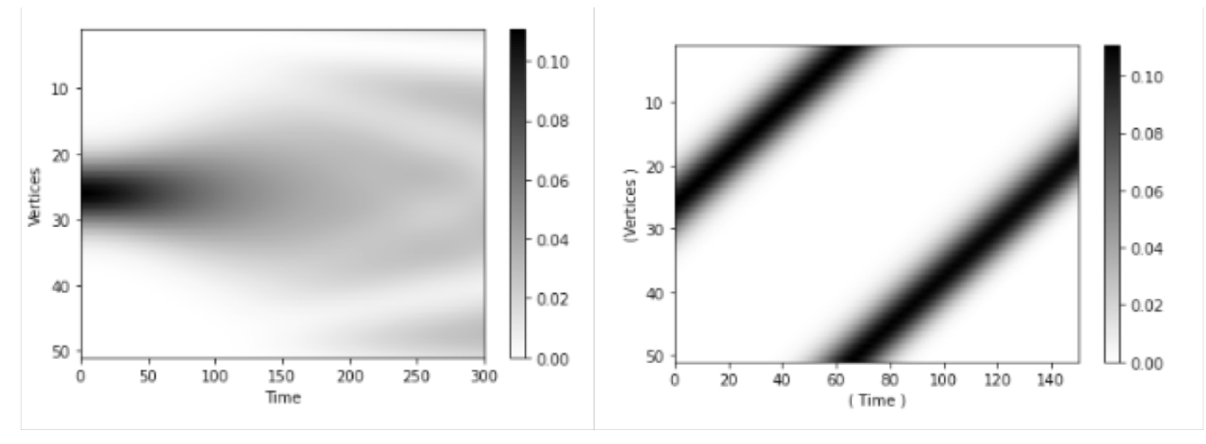

Figure 4: Walk on ring with initial Gaussian width=5, center=25, constant phase= 0 (left) & $\pi/2$ (right) for all nodes

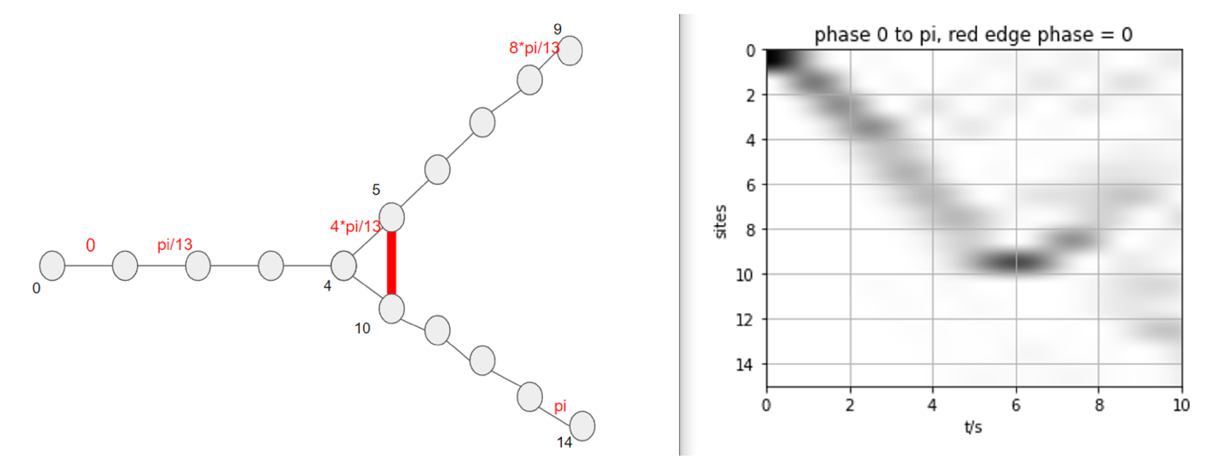

Figure 5: Linear edge weight ramp from 0 to π with red edge weight=0 (left) and its probability plot (right)

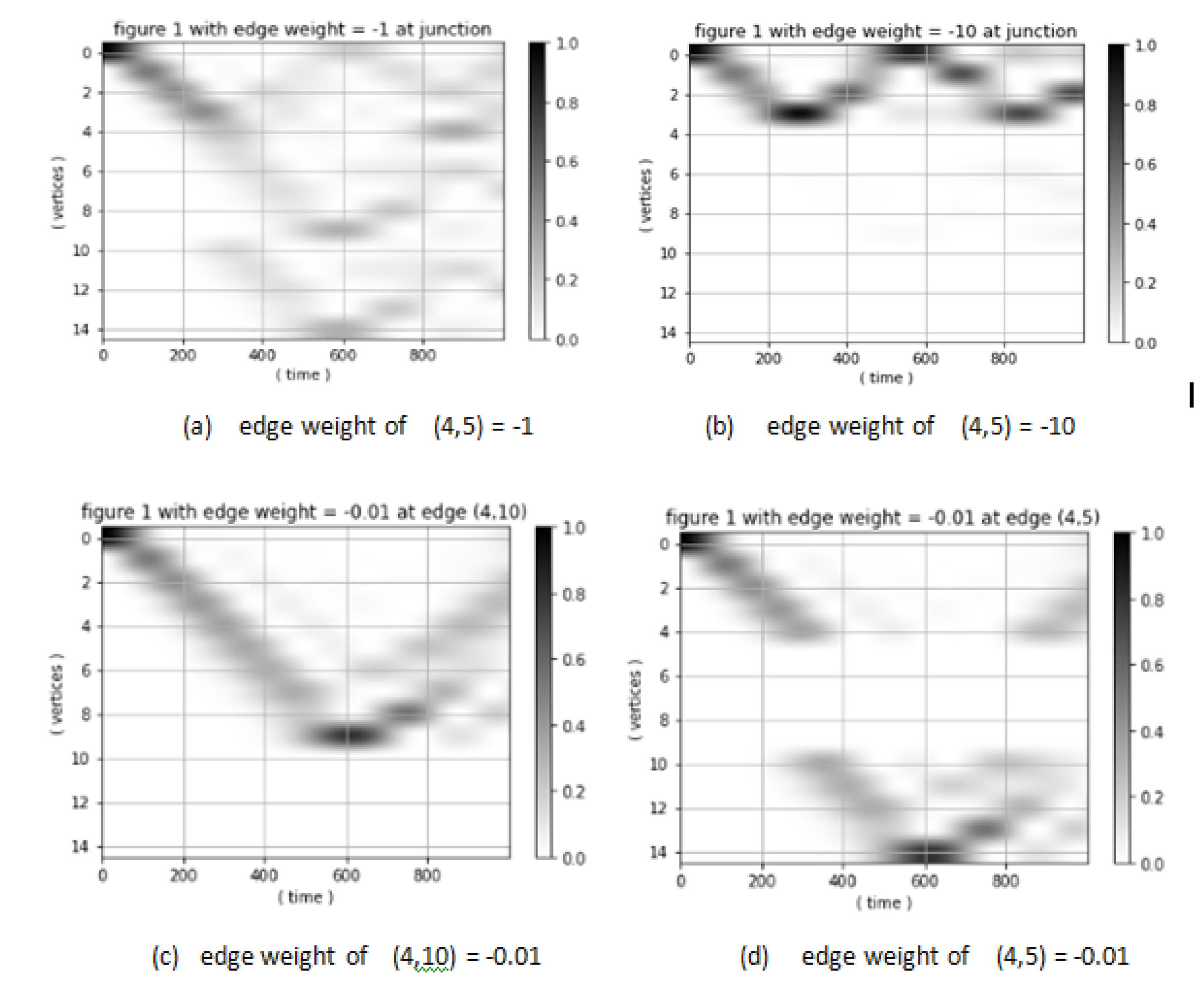

Figure 6: Walk on a branched graph with changing real edge weights, no complex phase

Results 3

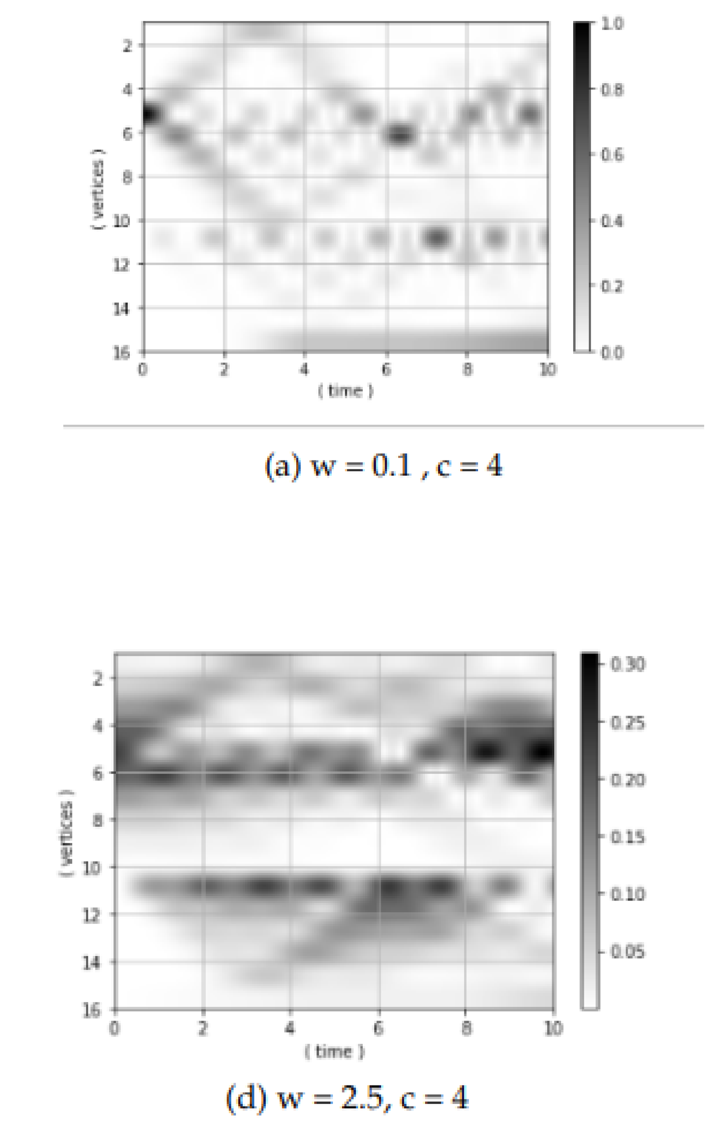

Figure 7: Walk on a branched graph with trap operator (site 16) connected with site 9, red edge phase = $-\pi/2$ and changing Gaussian width at center = 4, w = 0.1 (a) and 2.5 (d) , respectively

Conclusion

The directionality of the walk can be controlled through a number of parameters:

- Real and complex edge weights including phase ramps and constant weights
- Using initial Gaussian state or linear superposition of vertices
- Lindblad operators for open systems

Future Prospects

- Find optimum combination of the various methods mentioned.
- Develop better understanding of how the directionality is achieved using these methods.

References

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[3] Zimborás et al "Quantum Transport Enhancement by Time-Reversal Symmetry Breaking" Sci. Rep. 3 2361 (2013)
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